TIME-DEPENDENT QUEUEING APPROACH TO HELICOPTER ALLOCATION FOR FOREST FIRE INITIAL-ATTACK*†

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ABSTRACT

Helicopters are used extensively to transport initial-attack crews to forest fires in the province of Ontario. Each day fire managers must decide how to allocate the available helicopters to initial-attack bases. The helitack transport system at each base can be viewed as a multi-channel queue with customers (fires) and servers (helicopters). The authors describe a time-dependent queueing model of the helitack system and use numerical methods to estimate some of its operating characteristics. A dynamic programming model is then used to specify an optimal allocation of the available helicopters to helitack bases.

RÉSUMÉ

Des hèlicoptères sont employer souvent pour transporter les combattants d'attaque initiale aux incendies forestières dans la province de l'Ontario. Chaque jour les gérants d'opérations doivent décider comment attribuer les hèlicoptères disponibles aux bases. On peut envisage le système de transportation comme un système d'attente avec une ou plusiers chaînes (hèlicoptères) et clients (incendies). Les auteurs décrivent un modèle mathématique du système de transportation par hèlicoptères et ils utilisent les techniques numériques pour estimer quelques de ses caractéristiques d'opération. Un modèle de programmation dynamique est utiliser pour specifier une attribution optimal des hèli-coptères aux bases.

1 INTRODUCTION

The importance of early initial-attack has long been recognized by forest fire managers. The sooner an initial-attack crew begins control action on a fire, the more likely it will be extinguished at a small size. Because most forest fires in the province of Ontario are not readily accessible by land

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or water, fire managers were quick to appreciate the merits of airborne initial-attack. An abundance of lakes in forested areas, together with the development of float-equipped STOL aircraft like the Beaver and Otter, made it possible to build an effective airborne initial-attack system.

In recent years, there has been an increased use of helicopters for initial-attack transport purposes. The reduced landing area requirements of helicopters makes it possible for an initial-attack crew to land close to the fire and thereby reduce their ground travel time. The critical time interval between discovery and the start of initial-attack is thus decreased.

In this paper, we explore the possibility of using queueing theory and dynamic programming to develop decision-making aids to assist fire managers who must decide where to locate their helicopters each day. We focus on the transportation aspect of initial-attack using helicopters, upon a "queue" of fires whose arrival rates are described by time-dependent probabilities. Following Koopman,[11] we present a set of differential equations, the solution of which gives the probability that at a time t there are n fires in the queueing system. Numerical solution yields the time-dependent distribution of queue sizes.

The queueing results furnish the single-stage returns for a dynamic programming algorithm that can be used to specify an optimal allocation of helicopters to helitack bases. To illustrate the procedures, a numerical example based upon data from Northwestern Ontario is solved. We then discuss how the model might be refined.

2 THE INITIAL-ATTACK TRANSPORT SYSTEM

The province of Ontario is divided into a number of regions, each of which is further subdivided into several districts. In each district there are one or more helitack bases. Each point in a district is assigned to one of the helitack bases in order to create non-overlapping sectors, equal in number to the number of helitack bases.

A fire which occurs in a given sector is presumed to be served by a helicopter stationed at that sector's helitack base. Although this dispatching rule may seem overly restrictive, the distances between helitack bases are usually such that we believe this simplifying assumption to be reasonable.

Each morning the regional fire manager must decide how to allocate the available helicopters among the helitack bases within the region. We will assume that the number of available helicopters is such that at least one helicopter can be allocated to each base. We further assume that the helitack aircraft will be used solely for initial-attack transport purposes

and that the allocation made in the morning remains in effect for the entire day. Given these assumptions, the helitack transport system can be viewed as being comprised of a queueing system in each sector which operates independently of the other sectors in the region.

The queueing system in each sector can be envisaged as follows. Each time a fire is reported to the district dispatcher (customer arrives), the following sequence of events occurs:

1 The fire enters the initial-attack queue and awaits service.
2 As soon as a helicopter is available, it is used to service the fire which is waiting at the front of the initial attack queue. Service includes loading the crew, taking off, flying to the fire, scouting the fire, landing the crew, taking off again, flying back to the initial-attack base, landing and refuelling.
3 The helicopter is then available to transport another initial-attack crew to the next fire in the queue.

Using the standard terminology of queueing theory, the helitack system in each sector can be characterized as follows:

Arrival Process: The reporting of fires that require initial-attack is assumed to be a non-stationary Poisson process with the arrival rate depending upon the time of day.

Service Process: The service time distribution can be approximated by a stationary negative exponential distribution.

Queue Discipline: Since we are considering only initial-attack transport, it is reasonable to assume that the service discipline is First In, First Out.

System Capacity: Because the number of initial-attack crews available is finite, it is reasonable to limit the system capacity to some large number N. Sensitivity to the choice of N is discussed below.

Number of Channels: The number of service channels at a base is the number of initial-attack transport helicopters assigned to that base.

3 THE FIRE ARRIVAL PROCESS

Cunningham and Martell[5] showed that the number of man-caused fires that occur each day in a district can be represented by a Poisson distribution. It is reasonable to assume that the probability distribution of the number of lightning-caused fires that occur each day is also Poisson. Since the sum of independent Poisson-distributed random variates is also Poisson,[14] we can assume that the probability distribution of the number of fires that occur in a sector each day is Poisson.

Not all forest fires are detected on the same day that ignition occurs. Since initial-attack crews can only be dispatched to fires that have been reported, we need only consider the number of fires that are reported or

arrive each day. The term arrival is used to distinguish a reported fire from a fire occurrence (ignition). The fires that arrive on a given day include some fraction of those that occurred on that day, some fraction of those that occurred on the previous day, and so on. Since the Poisson distribution is preserved under random selection,[14] it is reasonable to assume that the probability distribution of the number of fires that arrive in a sector each day is Poisson.

Let F denote the expected number of forest fires that arrive in a particular sector on a given day. Because of diurnal weather variations and the characteristics of the detection system, the rate at which fires arrive varies during the day. We will assume that fires arrive according to a non-stationary Poisson process. Suppose the day is divided into T time periods and the length of period j is t_j hours. Let p_j denote the probability that a fire which arrives in the sector does so during period j. Let λ_j denote that fire arrival rate in the sector during period j, expressed in terms of fires per hour. We will suppose that λ_j can be estimated using the following equation:

$$\lambda_j = (p_j F)/t_j. \tag{3.1}$$

4 THE INITIAL-ATTACK TRANSPORT SERVICE PROCESS

On any given day, there are usually a number of different types of helicopters devoted to initial-attack transport. Most fire managers would prefer to use a large helicopter with the capacity to carry an entire five person unit crew and a standard load of helitack fire suppression equipment. To simplify our model, we will assume that the same type of helicopter is being used at all helitack bases within a single region.

The helitack transport service time for each fire includes the time required to load the helicopter, transport the initial-attack crew to the fire, return to the helitack base and refuel. The minimum service time will therefore exceed the time to load, take off, land, take off, land, and refuel. This time will, of course, vary, depending upon the alert status of the initial-attack crew, the fire boss's need to scout the fire and the ease of landing near the fire. Furthermore, a helitack crew would not be dispatched to fires that are located a short distance from the base. The service time would therefore be bounded from below by some non-zero time. Even though remote fuel caches are sometimes used, the transport service time is also limited to some upper bound, depending upon the helicopter's flying range and/or the sector boundary.

Although the actual service time distribution should be truncated above and below, we will assume that a standard negative exponential distribution is a satisfactory approximation. The mean service time will

be denoted by $1/\mu$, where the service rate of each helicopter is μ fires per hour. The number of service channels in each sector corresponds to the number of helicopters allocated to it for the day. However, only one queue will operate at each base, with service provided by the first available helicopter.

5 THE DISTRICT HELITACK QUEUEING SYSTEM

5.1 Introduction

Consider a queueing system for which the arrival rate $\lambda(t)$ and service rate $\mu(t)$ are time-dependent. That is, in the case of Poisson arrivals, $\lambda(t)\Delta t$ is the probability that between times t and $(t + \Delta t)$ a customer joins the queue. Similarly, if the service distribution is negative exponential, $\mu(t)\Delta t$ is the probability that a customer has finished service by time $(t + \Delta t)$, given that at time t service has already begun.

Until a short time ago, the quantitative analysis of a time-dependent queueing system was limited either to computer simulation or to treatment as a Markov process possessing steady-state transition probabilities. The former approach often entails costly computer expenditures. The steady-state treatment, on the other hand, is often inappropriate when, as is the case with forest fire initial-attack transport, customer arrival rates are time dependent. To study the transient behaviour of such queues, it is clear that a fully time-dependent approach is desirable.

The first treatment of the $M/M/1$ queueing system with time-varying parameters $\lambda(t)$ and $\mu(t)$ was by Clarke.[4] The equations for a birth and death process (see, for example, Saaty[16] were solved analytically to yield complicated expressions for $P_n(t)$, the probability that at time t there are n customers in the queueing system. This approach, based upon an integral equation for the generating function of $P_n(t)$, is a suitable starting point for a calculation if analytical expressions are available for both $\lambda(t)$ and $\mu(t)$.

Often, however, one has available only numerical estimates for $\lambda(t)$ and $\mu(t)$. In such a situation, it is computationally advantageous to numerically solve the birth and death equations directly. This was the approach taken by Koopman.[11] Assuming time-dependent Poisson arrival rates $\lambda(t)$, and treating the queue evolution as Markovian, he derived a system of weakly coupled stochastic differential equations for $P_n(t)$. This system differs from the usual set of forward or backward equations[16] in that there is an upper limit of N customers which may be in the queueing system at any given time (i.e.; for all t, $P_n(t) = 0$ for $n > N$). Using for $\lambda(t)$ actual flight statistics concerning landings at Kennedy and LaGuardia airports in New York, Koopman[11] obtained a numerical solution of the set of equations.

Koopman[11] also briefly treated the case of *multiple* queues, in which the same runway (i.e. same "server") is used for both the arrivals and departures of aircraft. He derived, but studied only qualitatively, an analogous system of $(N + 1)(M + 1)$ differential equations for $P_{nm}(t)$, the probability that at time t there are n aircraft in the air awaiting landing clearance and m in the ground queue awaiting takeoff. This system of equations for multiple queues of aircraft under time-dependent conditions has been solved by Bookbinder and Luthra[2] (see also Bookbinder[1]), and policy options presented for Toronto International Airport.

Similar approaches to air transportation based upon Koopman's work have been used by Hengsbach and Odoni[9] to estimate the time-dependent marginal cost due to aircraft delay, and by Bundy and Giffin[3] in a study of air traffic-control communications as a time-varying queue with feedback. Kolesar et al.[10] have used Koopman's equations to estimate the transient delays of urban police cars in responding to alarms. Moore[13] has published an imbedded Markov chain analysis of non-stationary single-server queues.

It should be noted that in the preceding references and in the approach of the present paper, a time-dependent approach was used because the arrival rate was not constant, but varied significantly throughout the period of interest. Consequently, there is no question of using a steady-state solution; the latter may not exist and in any case it is not meaningful. Numerical solution of the birth-and-death equations is required to find $P_n(t)$.

On the other hand, even for constant λ and μ, one may be interested in the transient solution for $P_n(t)$. This is, of course, the solution for "small times," before the steady-state values P_n, independent of time, are reached. Recent work by Grassmann[7] has shown how to compute numerically such transient solutions efficiently.

5.2 *Time-dependent equations for an S-server queue*

The queueing aspects of our initial-attack transport model are based upon the work of Koopman[11] and the numerical work of Bookbinder.[1] Consider a sector in which there are S initial-attack transport helicopters. The appropriate generalization of Koopman's equations for the case of S helicopters is (Kolesar at al.[10]):

$$n = 0 \quad dP_0/dt = -\lambda(t)P_0(t) + \mu P_1(t),$$

$$1 \leqslant n < S < N \quad dP_n/dt = -(\lambda(t) + n\mu)P_n(t) + \lambda(t)P_{n-1}(t) + (n + 1)\mu P_{n+1}(t),$$

$$1 \leqslant S \leqslant n < N \quad dP_n/dt = -(\lambda(t) + S\mu)P_n(t) + \lambda(t)P_{n-1}(t) + S\mu P_{n+1}(t),$$

$$n = N \quad dP_N/dt = -S\mu P_N(t) + \lambda(t)P_{N-1}(t),$$

TABLE 1

FREQUENCY DISTRIBUTION OF FOREST FIRE
ARRIVAL TIMES

Time Interval	Relative Frequency	Cumulative Relative Frequency
5–6	.0000	.0000
6–7	.0186	.0186
7–8	.0139	.0325
8–9	.0162	.0487
9–10	.0348	.0835
10–11	.0580	.1415
11–12	.1206	.2622
12–13	.1369	.3991
13–14	.0951	.4942
14–15	.1253	.6195
15–16	.1508	.7703
16–17	.0998	.8701
17–18	.0394	.9095
18–19	.0348	.9443
19–20	.0232	.9675
20–21	.0162	.9838
21–22	.0139	.9977
22–23	.0023	1.0000
23–24	.0000	1.0000

where $P_n(t)$ is the probability that at time t, there are n customers in the sector queueing system, including, of course, any that are presently being served. N, which is the sector queueing system capacity, will be discussed below.

6 NUMERICAL SOLUTION IN THE TRANSIENT CASE

6.1 *Numerical values of parameters*

The system of differential equations governing the evolution of the queueing system (hereafter referred to as (I)) requires the specification of $\lambda(t)$, μ, N and the initial conditions $P_n(t_0)$, $n = 0, 1, 2, \ldots N$. Note that we have specialized to the case $\mu(t) = \mu$ for all t.

For the example considered in this paper, the arrival rate $\lambda(t)$ is based on an analysis of 430 fires which occurred in the northwestern region of Ontario during the summers of 1967 through 1969.[12] The variation of the fire arrival rate throughout the day can be represented by an empirical probability function p_j (see table 1) which for $j = 1, 2, 3, \ldots 19$ represents the relative likelihood of a fire being detected during the discrete time interval j. ($j = 1$ refers to the period 5 A.M. to 6 A.M., $j = 19$ to the time between 11 P.M. and midnight.)

It is reasonable to assume that the relative likelihood of fires being detected during different periods of the day is the same for that time period every day. Accordingly, if t_j equals 1 hour, the arrival rate $\lambda(t)$ is $p_j F$. Here, F is a scale factor which represents the expected number of fires that arrive on the given day.

6.2 Service rates

Our service time distribution is based upon data from the Red Lake district of Northwestern Ontario.[15] Thirty-five helitack sorties were assumed to be representative of the helitack operation of the Ontario Ministry of Natural Resources. The service times varied from 0.3 hours to a maximum of 3.7 hours. The mean service time of 1.53 hours corresponds to a service rate μ of 0.65 fires per hour.

It should be noted that for the 35 sorties in question, the standard deviation σ of service times is only 0.74 hours. σ/μ is thus approximately $1/2$, whereas for a negative exponential distribution, $\sigma = \mu$. Moreover, it would be unusual to observe a service time less than 0.2 hr or greater than about 4.0 hr. (The helitack "zone of influence" for most helicopters is usually less than 100 miles.) These remarks concerning σ and μ cast doubt upon the service times as a sample from a negative exponential distribution.

Our suspicion that the service time distribution is not negative exponential was confirmed by the results of goodness-of-fit tests. We used three tests suggested by Gross and Harris[8]: the F test, the Kolmogorov-Smirnov test, and the Anderson-Darling test. In all three cases, the null hypothesis that the service time distribution is negative exponential was rejected at the $\alpha = .01$ level of significance.

Nevertheless, Koopman[11] showed that the calculations of averages, such as expected number in the queue and the mean waiting times, are for an $M/G/1$ queue remarkably insensitive to whether or not the service distribution is negative exponential (i.e., completely "random") or even constant (deterministic). Following Koopman[11] we argue that these two extremes must surely bracket the actual situation in cases when one is primarily interested in the calculation of system averages. Accordingly, we employed the set of equations (I), which requires a negative exponential service distribution, with μ given by its average value of 1.53 hours.

6.3 Maximum number in queueing system

Were it not for the imposition of the maximum queue size N ($P_n(t) = 0$ for $n > N$), the number of equations in the system would be infinite. In the steady state, these infinitely many equations are easily solved recursively in terms of the ratio $\rho = \lambda/\mu$. For the transient case, however, one must work with a finite system.

The choice of N is dictated by numerical considerations. One begins with an initial choice, solves the system (I) for $P_n(t)$, and examines the values of $P_N(t)$ for times during the busy period. Even during the hours of peak activity, for properly chosen N, the magnitude of $P_N(t)$ should approach zero. If this is not the case, N must be increased and numerical solution repeated.

6.4 *Numerical procedures*

Initiation of numerical solution of the differential equations requires specification of the probabilities $P_n(t_0)$. Since the integration begins at $t_0 = 5$ A.M., the first thought is to take

$$P_0(5) = 1, \tag{6.4.1}$$

with $P_n(5) = 0$ for $n \geqslant 1$. This corresponds to an initially empty queueing system, and is consistent with the frequency distribution of forest fire arrival times (table 1).

However, for times of 8 P.M. and later, it is usually too dark for dispatch to be initiated. This required elimination of the terms involving μ from the set of equations (I). Even though λ_j is small between 8 P.M. and midnight, the queue begins to lengthen, since μ is effectively zero during this time. Thus, for a sequence of days in which the relative probabilities are given by table 1, $P_n(5$ A.M.$)$ is not δ_{n0}, where δ_{n0} is 1 if n equals zero, and is zero otherwise. Rather, $P_n(5$ A.M.$)$ is given by the probability distribution of fires which prevailed on the previous midnight.

Use of the latter distribution as the initial condition only effects the calculated $P_n(t)$ for the first hour or two the following day (with the length of this transient period depending upon F and S). After this period the $\{P_n(t)\}$ are identical when calculated from either set of starting values. Since we were interested in the maximum queue length, which does not occur in either case until 3 or 4 P.M., for convenience we have used δ_{n0} as the initial set $\{P_n(5)\}$.

With (6.4.1) as initial conditions and the above modification for μ, numerical integration proceeded by first smoothing the arrival rate $\lambda(t)$. That is, to remove the "artificial" divisions caused by hourly boundaries, we took for the discrete units of time $j = 1, 2, 3, \ldots$, the moving average

$$\bar{\lambda}_j = (\lambda_{j-1} + \lambda_j + \lambda_{j+1})/3.$$

End-point corrections at times 5.0 hr and 24 hr were chosen such that the sum of the expected numbers of fires was unchanged for the day as a whole. This smoothed arrival rate was used as coefficients in (I).

Numerical solution of the differential equations was accomplished through a double-precision, Runge-Kutta approach based upon the

fourth-order formulas of Fehlberg.[6] A variable step size Δt was used, in order to keep the local error per unit step within a tolerance of 0.1%. Where necessary, a linear interpolation was performed between successive values of the already smoothed $\bar{\lambda}$.

The result of the integration is thus, for each value of S and F, the distribution $P_n(t)$ throughout the day. From this distribution, we calculated

$$\max_t L_q(S, t),$$

where L_q is the expected number of fires actually awaiting service. This number is used as input to the dynamic programming algorithm for helicopter allocation, which is discussed in the following section.

7 THE HELICOPTER ALLOCATION MODEL

Each morning, the regional fire manager must decide how to allocate the available helicopters among the helitack bases within his region. Since the primary objective of a helitack system is to deliver an initial-attack crew to the fire as quickly as possible, he will, of course, make an allocation so as to minimize congestion in the queues. Since not all sectors are of equal value, the manager will also likely attempt to take relative damage potential into account.

Suppose the parameter v_k denotes the relative fire damage that might be incurred if an acre of forest in sector k is burned, compared to an acre in another sector. Without loss of generality, we can assume that v_k ranges from 0 to 1. In other words, if v_k equals .2 and v_j equals .4, we are assuming that the penalty for delaying initial attack on a fire in sector j is twice that in sector k.

We believe that most regional fire managers would be satisfied with an objective of minimizing the maximum expected queue length, summed over all sectors and weighted by the damage parameters v_k. Such an objective would represent a desire to minimize congestion while taking the relative values of different sectors into account.

Assuming that at least one helicopter is allocated to each of B helitack bases and letting X_k denote the number of helicopters allocated to base k, the allocation problem can be formulated as a simple dynamic programming problem with the following recursion relationship.

$$f_k^*(S_k) = \min_{1 \leqslant X_k \leqslant S_k - (B-k)} \left(V_k \max_t L_q(X_k, t) + f_{k+1}^*(S_k - X_k) \right),$$

where S_k denotes the number of helicopters available at the start of stage k.

TABLE 2

SECTOR FIRE LOAD

Sector	F	v_k
1	10	.2
2	5	.6
3	3	.3

TABLE 3

MAXIMUM EXPECTED QUEUE LENGTHS

Number of helicopters	Base		
	1	2	3
1	3.2612	.8968	.3109
2	1.0911	.1627	.0367
3	.3163	.0270	.0042
4	.0834	.0040	.0004

8 EXAMPLE

As a numerical example, consider the allocation of six helicopters among three helitack bases. The average number of fires per day and the damage factors are shown in table 2.

The queueing model described in Section 5 was used to estimate the maximum expected queue lengths that would occur in each sector, depending upon the number of helicopters allocated there. These results are shown in table 3. (N.B. These refer to the expected numbers of fires in the queue and do not include any fires presently being serviced.)

Using these estimates and backward induction, one can derive the optimal allocation of helicopters shown in table 4.

9 DISCUSSION

We wish to point out that the model we have presented could also be used to help decide where to locate land-based fire retardant bombing aircraft. The use of such aircraft is often governed by the "one strike" concept. That is, they make only one sortie to a fire. The service time is therefore independent of the length of time the fire waits in the queue. The only major difference is that usually more than one retardant bomber is dispatched to each fire. It would be necessary to assume that this number is the same for all fires and that together they constitute one server.

TABLE 4

OPTIMAL HELICOPTER ALLOCATION

Base	Number of helicopters
1	3
2	2
3	1

In the preceding pages we have described a model which we believe captures the essence of the initial-attack transport helicopter allocation problem. It has, of course, been necessary to make a number of simplifying assumptions that should be explored more fully in future research efforts. For example, although a FIFO queue discipline was assumed, some fires, particularly those threatening high value areas, are dealt with on a priority basis. The helitack operation at each base was assumed to operate independently of all other bases in a region. Although this is usually the case, fire crews are sometimes dispatched to fires in neighbouring sectors.

While we believe that our model can be used to help forest fire managers, further research should be undertaken to develop an expanded model that incorporates more of the complexity of the helitack planning problem. For example, the model should be expanded to include various modes of transport, including different types of helicopters, fixed-wing aircraft, and road travel. This would of course entail a detailed analysis of dispatching rules, an important aspect of fire management which is beyond the scope of our present model. Our hope is that future developments in queueing theory will make it possible to incorporate these and other aspects of initial attack planning into an improved fire management decision-making aid.

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